Inventory Management with Advance Capacity Information

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One of the important aspects of supply chain management is finding ways to circumvent the negative effects of demand and supply uncertainty. The uncertainty of future supply can be lowered, if a supply chain member is able to obtain advance capacity information (ACI) on future supply/production capacity availability from its supplier. We address a periodic-review inventory system under stochastic demand and stochastic limited supply, for which ACI is available. We show that the optimal ordering policy is a state-dependent base stock policy characterized by a base stock level, which is a function of ACI. We establish a link to advance demand information (ADI) inventory research stream by developing a capacitated inventory system with ADI, and showing that the model is closely related to the proposed ACI model. We perform a numerical analysis to quantify the value of ACI and obtain useful managerial insights from the study of the specific problem instances.

Keywords: Supply Chain Management, Stochastic Inventory Control, Base Stock Policy, Advance Capacity Information

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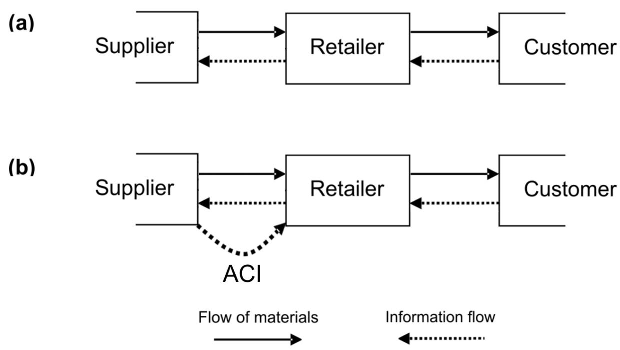
1. Introduction

Foreknowledge of future supply availability is useful in managing an inventory system. Anticipating possible future supply shortages is beneficial to make timely ordering decisions, which results in either building up stock to prevent future stockouts, or reducing the stock in the case the supply conditions in the future might be favorable. Thus, system costs can be reduced by carrying less safety stock while still achieving the same level of performance. These benefits should encourage the supply chain parties to formalize their cooperation to enable the necessary information exchange. We can argue that extra information is always beneficial, but further thought has to be put into investigating in which situations the benefits of information exchange are substantial and when it is only marginally useful. While in the first case it is likely that the benefits will outweigh the cost related to adopting the information sharing system, in the latter these costs are not justified. The need to establish long-term cooperation enabling the information exchange is particularly strongly motivated by the recent increasing move to outsource production and other activities to contract manufacturers. To minimize the risk of contract manufacturing agreements failing to live up to expectations, companies put effort into managing the relationship with their vendors. The goal is that vendor tailors his services to each customer's specific needs, and provide accurate lead times and delivery dates. To do so, he is willing to share lead time and capacity information with his customers.

In this paper, we study the benefits of obtaining advance capacity information (ACI) about future uncertain supply capacity. The scope is on a single member of a supply chain, where the generality of the problem is such that this member can be either a manufacturer (production company), which itself produces the product, as well as a distributer or a retailer, which raises its inventory position by placing orders to its predecessor in the supply chain. These benefits are assessed based on the comparison between the case where a supply chain member, for instance a retailer, is able to obtain ACI from its supplier, and a base case of no information. We characterize a traditional supply chain as a supply chain where ACI is not shared. Due to stochastic supply conditions the retailer is uncertain about the actual order size that will be delivered. This uncertainty can be due to the changing availability of supply delegated to the particular retailer by its supplier and/or due to overall product unavailability at certain times. In a production setting the capacity is determined by productive resources, such as machinery or workforce. The ability to produce a product in desired quantities strongly depends on the availability of these resources. Stochastic nature of capacity changes may be due to variations in the workforce (e.g. holiday leaves), unavailability of machinery due to short-term planned maintenance or due to multiple products sharing the total capacity. It is reasonable to assume that some of these variations can be foreseen. At least in a near future the supplier can be certain about the extent of the capacity that he can delegate to a particular retailer. Similarly, in a production setting a short-term production plan is fixed and the uncertainty in the number of the available workforce is lower in a near future (workers report their holiday plans in advance). Since the supplier or manufacturer has insight into future capacity availability, he can communicate ACI to the retailer (Figure 1), and thus help her to reduce the supply uncertainty and consequently lower the inventory cost.

Figure 1 (a) Traditional supply chain and (b) Supply chain with ACI sharing.



To build the foundation for our work, we proceed with a brief review of the relevant literature. Although the uncapacitated problems form a foundation in the stochastic inventory control research field, we are interested in inventory models that simultaneously tackle the capacity that may limit the order size or the amount of products that can be produced. These models not only recognize that the supply chain's demand side is facing uncertain market conditions, but also look at the risks of limited or even uncertain supply conditions. The researchers revisited the early stochastic demand models and extended them to incorporate the uncertainty on the supply side. As we proceed with the literature review on capacitated inventory problems, it becomes clear that the base stock policy characterizes an optimal policy for several different capacitated problems. The sense of a base stock policy is different in the resource constrained case than in the uncapacitated case. In the uncapacitated case, the base stock level has a clear interpretation, it is the inventory position to order/produce-up-to. In the capacitated case, however, it only represents a target that may or may not be achieved. If the capacity limit in a certain period is known, there is no use in ordering/producing above that level, thus, we are talking about a modified base stock policy. Federgruen and Zipkin (1986a,b) first address the fixed capacity constraint for stationary inventory problem and prove the optimality of the modified base stock policy. This result is extended by Kapuscinski and Tayur (1998) for the non-stationary system assuming periodic demand, where they also show that a modified base-stock policy is optimal. Later, a line of research extends the focus to capture the uncertainty in capacity, by analyzing a limited stochastic production capacity models (Ciarallo et al., 1994; Güllü et al., 1997; Khang and Fujiwara, 2000; Iida, 2002). Here we point out the relevance of Ciarallo et al. (1994) to our work. In the analysis of a single period problem, they show that stochastic capacity does not affect the order policy. The myopic policy of newsvendor type is optimal, meaning that the decision maker has no incentive to ask for the quantity higher than that of the uncapacitated case. For the finite horizon stationary inventory model they show that the optimal policy remains to be a base stock policy, where the optimal base stock level is increased to account for the possible, however uncertain, capacity shortfalls in the future periods. Our ACI inventory model builds on these models assuming both demand and capacity are non-stationary and stochastic. The extension we are proposing is that by using ACI we can lower the supply capacity uncertainty and better align the optimal policy parameters with revealed supply capacity realizations in near future periods. To our knowledge the proposed way of modeling inventory system with ACI has not yet received any attention in the research community.

The complexity of a capacitated stochastic non-stationary inventory problem presents a too hard challenge in terms of obtaining analytical solution for the parameters of the optimal policy, mainly a period-to-period optimal base stock level. The researchers resorted to developing applicable heuristics (Bollapragada and Morton, 1999; Metters, 1997, 1998). In a non-stationary stochastic setting, the models are capturing both the effect of deterministic anticipation (anticipating mismatches in demand and capacity, and reacting by building up the inventory), as well as the effect of uncertainty in demand. Again, we note that there is a lack of literature on approximate analysis of inventory systems, which assume uncertain capacity as well.

We propose that an effective way of circumventing the uncertainty of supply capacity is by obtaining ACI. This logic can also be targeted at the demand side, and is dealt by the already well established advance demand information (ADI) research stream (Gallego and Özer, 2001; Karaesmen et al., 2003; Wijngaard, 2004; Tan et al., 2007). Usually it is assumed that the future uncertainty can be reduced due to some customers that place their orders in advance of their needs. This forms the stream of early demand that does not have to be satisfied immediately. Since this demands is revealed beforehand through ADI, we can use ADI to make better ordering decisions. For the capacitated inventory model with ADI, Özer and Wei (2004) show that the base stock policy is optimal, where the optimal base stock level is an increasing function of the size of ADI. In terms of modeling, our approach resembles ADI modeling approach, therefore our work also focuses on presenting the possible similarities and the relevant distinctions between the two.

Our contributions in this study can be summarized as follows: (1) We develop a stochastic inventory model, which through the use of ACI, potentially enables the decision maker to improve the performance of the inventory control system, (2) we demonstrate useful structural properties of the optimal policy, (3) we come up with the corresponding ADI inventory model and comment on its characteristics in relationship to the proposed ACI model, (4) our computational results provide useful managerial insight into circumstances where ACI becomes most beneficial.

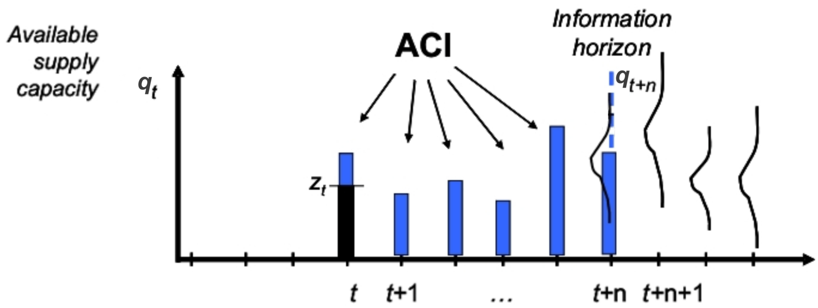
The remainder of the paper is organized as follows. We present a model incorporating ACI and its dynamic cost formulation in Section 2. The optimal policy and its properties are discussed in Section 3, where we also look at the similarities between the ADI modeling and the analysis of ACI model presented in this paper. In Section 4 we present the results of a numerical study and point out additional managerial insights. Finally we summarize our findings and suggest directions for future research in Section 5.

2. Model Formulation

In this section, we introduce the notation and our model. The model under consideration assumes periodic-review, non-stationary stochastic demand, limited non-stationary stochastic supply with zero supply lead time, finite planning horizon inventory control system. For the sake of generality of the model, we discuss the case of a fixed nonnegative supply lead time in Section 2.1. However, the manager is able to obtain ACI on the available supply capacity for the orders to be placed in the future and use it to make better ordering decisions. We introduce a parameter n, which represents the length of the ACI horizon, that is, how far in advance the available supply capacity information is revealed. We assume ACI q_{t+n} is revealed at the start of period t for the supply capacity that limits the order z_{t+n}, that will be placed in period $t+n$. The model assumes perfect ACI, which means that in period t the exact supply capacities limiting the orders that will be placed in the current and the following n periods are known. The supply capacities in the more distant periods, from $t+n+1$ towards the end of planning horizon, remain uncertain (Figure 2). This means that when placing the order z_t in period t, we know the capacity limit q_t, and it is rational to order up to this limit only.

Assuming that unmet demand is fully backlogged, the goal is to find an optimal policy that would minimize the relevant costs, that is inventory holding costs and backorder costs. We

Figure 2 **Advance capacity information.**



assume zero fixed cost inventory system. The model presented is general due to the fact that no assumptions are made with regards to the nature of demand and supply process. Both are assumed to be stochastic and with known independent distributions in each time period. The major notation is summarized in Table 1 and some is introduced when needed.

Table 1 **Summary of Notation**

T	:	number of periods in the finite planning horizon
L	:	constant nonnegative supply lead time, a multiple of review periods ($L \geq 0$);
n	:	advance capacity information, $n \geq 0$
h	:	inventory holding cost per unit per period
b	:	backorder cost per unit per period
α	:	discount factor ($0 \leq \alpha \leq 1$)
x_t	:	inventory position at time t before ordering
y_t	:	inventory position at time t after ordering
$\hat{x}_t$	:	starting net inventory at time t
z_t	:	order size at time t
D_t	:	random variable denoting the demand in period t
d_t	:	actual demand in period t
g_t	:	probability density function of demand in period t
G_t	:	cumulative distribution function of demand in period t
Q_t	:	random variable denoting the available supply capacity at time t
q_t	:	actual available supply capacity at time t, for which ACI was revealed at time $t-n$
r_t	:	probability density function of supply capacity in period t
R_t	:	cumulative distribution function of supply capacity in period t

We assume the following sequence of events. (1) At the start of the period t, the decision maker reviews the current inventory position x_t and ACI on the supply capacity limit q_{t+n}, for the order z_{t+n} that is to be given in period $t+n$, is revealed. (2) The ordering decision z_t is made based on the available supply capacity q_t, where $z_t \leq q_t$, and correspondingly the inventory position is raised to $y_t = x_t + z_t$. (3) The order placed at period $t-L$ is received. (4) At the end of the period previously backordered demand and demand d_t are observed and satisfied from on-hand inventory; unsatisfied demand is backordered. Inventory holding and backorder costs are incurred based on the end-of-period net inventory.

To determine the optimal cost, we not only need to keep track of x_t, but also the supply capacity available for the current order q_t, and the supply capacities available for future orders, which constitute ACI. At the start of the period t, when the available supply capacity q_{t+n} is already revealed for period $t+n$, the vector of ACI consists of available supply capacities potentially limiting the size of the orders in the future n periods, $\vec{q_t} = (q_{t+1}, q_{t+2}, \ldots, q_{t+n-1}, q_{t+n})$. The information on the current supply capacity q_t obviously affects the cost, but need not be included in the ACI vector, since, as we show in Section 3 and 3.1 in particular, only ACI for future orders affects the structure and the parameters of the optimal policy. All together, the state space is represented by an $n+2$ -dimensional vector $(x_t, q_t, \vec{q_t})$, where x_t and $\vec{q_t}$ are updated at the start of period $t+1$ in the following manner

$$\begin{aligned} x_{t+1} &= x_t + z_t - d_t, \\ \vec{q}_{t+1} &= (q_{t+2}, q_{t+3}, \ldots, q_{t+n+1}). \end{aligned} \tag{1}$$

Observe that in the case of $n=0$, the supply capacity information affecting the current order is revealed just prior to the moment when the order needs to be placed. In this specific setting the state space is reduced to 2-dimensional, where we only need to track the changes in x_t, and place an order accordingly to the currently available supply capacity q_t.

Our model excludes the situation where the order is placed not knowing to what extent it will be fulfilled. In this case we are talking about the capacitated stochastic supply model with no ACI, the model that has been introduced by Ciarallo et al. (1994). However, when assuming $n=0$ under the zero lead time setting ($L=0$), we show that under this specific setting optimal costs under both models are equivalent (Section 3.1).

The minimal discounted expected cost function, optimizing the cost over a finite planning horizon T from time t onward and starting in the initial state $(x_t, q_t, \vec{q_t})$, can be written as:

$$\begin{aligned} f_t(x_t, q_t, \vec{q_t}) = \min_{x_t \le y_t \le x_t + q_t} \{C_t(y_t) + \\ + \begin{cases} \alpha E_{D_t}[f_{t+1}(y_t - D_t, q_{t+1}, \vec{q}_{t+1})]\}, & \text{if } T-n \le t \le T, \\ \alpha E_{D_t, Q_{t+n+1}}[f_{t+1}(y_t - D_t, q_{t+1}, \vec{q}_{t+1})]\}, & \text{if } 1 \le t \le T-n-1, \end{cases} \end{aligned} \tag{2}$$

where $C_t(y_t) = h\int_0^{y_t}(y_t - d_t)g_t(d_t)\mathrm{d}d_t + b\int_{y_t}^{\infty}(d_t - y_t)g_t(d_t)\mathrm{d}d_t$ is the regular loss function, and the ending condition is defined as $f_{T+1}(\cdot) \equiv 0$.

2.1. Supply lead time considerations

In this section we extend the zero supply lead time model to account for a more realistic situation of positive supply lead time. Since each of the orders remains in the pipeline stock for L periods, we can express the inventory position before ordering x_t as the sum of net inventory and pipeline stock,

$$x_t = \hat{x}_t + \sum_{s=t-L}^{t-1} z_s. \tag{3}$$

Correspondingly, the inventory position after ordering is $y_t = x_t + z_t$, where z_t is limited by q_t. Note, that due to perfect ACI the inventory position y_t at all times reflects the actual quantities that will be delivered in the current and the following L periods.

Due to constant non-zero lead time the decision maker should protect the system against lead time demand, $D_t^L = \sum_{s=t}^{t+L} D_s$, which is demand realized in the time interval $[t, t+L]$. Since the current order z_t affects the net inventory at time $t+L$, and no later order does so, we reassign the corresponding expected inventory-backorder cost $C_t(y_t)$ to period t, by the L-period discount factor α^L. The single period cost function $C_t(y_t)$ is again the regular loss function as in (2), where this time the expectation is with respect to lead time demand D_t^L. The form of the optimal cost function remains as given in (2).

Here we should point out that the presented ACI model assumes that ACI reveals the supply capacity availability for the current and n future orders, that will be delivered in periods $t+L$ to $t+L+n$. Meaning that when placing the order exact supply capacity realization is known. In Jakšič (2008) we discuss the possible extension to the presented ACI model, where we assume that supply information is received only after the order has been placed. In this case the order has to be placed not knowing the available supply capacity, however, we observe *advance supply information* for the order that is already given and is currently still in the pipeline. Advance supply information indicates whether order will be filled fully or just partially before the actual delivery, and thus enables the decision maker to react if necessary.

3. Analysis of the Optimal Policy

In this section, we first characterize the optimal policy, as a solution of the dynamic programming formulation given in (2). We prove the optimality of a state-dependent modified base stock policy and provide some properties of the optimal policy. For proofs of the following theorems, see Appendix.

Let J_t denote the cost-to-go function of period t defined as

$$J_t(y_t, \vec{q}_t) = \begin{cases} C_t(y_t) + \alpha E_{D_t}[f_{t+1}(y_t - D_t, q_{t+1}, \vec{q}_{t+1})], & \text{if } T-n \le t \le T, \\ C_t(y_t) + \alpha E_{D_t, Q_{t+n+1}}[f_{t+1}(y_t - D_t, q_{t+1}, \vec{q}_{t+1})], & \text{if } 1 \le t \le T-n-1, \end{cases}$$

and we rewrite the minimal expected cost function f_t as

$$f_t(x_t, q_t, \vec{q}_t) = min_{x_t \le y_t \le x_t + q_t} J_t(y_t, \vec{q}_t), \quad \text{for} \quad 1 \le t \le T.$$

We first show the essential convexity results that allow us to establish the optimal policy. Note that the single period cost function $C_t(y)$ is convex in y, since it is the usual newsvendor cost function (Porteus, 2002).

THEOREM 1. *For any arbitrary value of the information horizon n and value of the ACI vector $\vec{q}$, the following holds for all t:*

1. *$J_t(y, \vec{q})$ is convex in y,*
2. *$f_t(x, \vec{q})$ is convex in x.*

Based on convexity results, minimizing J_t is a convex optimization problem for any arbitrary ACI horizon parameter n.

THEOREM 2. *Let $\hat{y}_t(\vec{q}_t)$ be the smallest minimizer of the function $J_t(y_t, \vec{q}_t)$. For any $\vec{q}_t$, the following holds for all t:*

1. *The optimal ordering policy under ACI is a state-dependent modified base stock policy with the optimal base stock level $\hat{y}_t(\vec{q}_t)$.*
2. *Under the optimal policy, the inventory position after ordering $y_t(x_t, q_t, \vec{q}_t)$ is given by*

$$y_t(x_t, q_t, \vec{q}_t) = \begin{cases} x_t, & \hat{y}_t(\vec{q}_t) \le x_t, \\ \hat{y}_t(\vec{q}_t), & \hat{y}_t(\vec{q}_t) - q_t \le x_t < \hat{y}_t(\vec{q}_t), \\ x_t + q_t, & x_t < \hat{y}_t(\vec{q}_t) - q_t. \end{cases}$$

This modified base stock policy is characterized by a state-dependent optimal base stock level $\hat{y}_t(\vec{q}_t)$, which determines the optimal level of the inventory position after ordering. The optimal base stock level depends on the future supply availability, that is supply capacities $q_{t+1}, q_{t+2}, \ldots, q_{t+n}$, given by ACI vector.

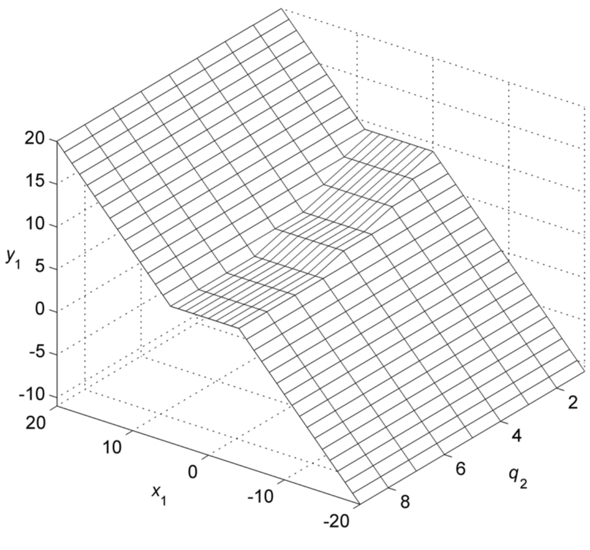
It is important to note here that since the optimal base stock level is the smallest minimizer of the cost-to-go function $J_t(y_t, \vec{q}_t)$, it does not depend on the supply capacity q_t, as J_t is not a function of q_t. However the actual realization of y_t is restricted by the supply capacity q_t available in period t.

Remark 1. *The optimal base stock level $\hat{y}_t(\vec{q}_t)$ is independent of the available supply capacity q_t in period t.*

The optimal policy can thus be interpreted in the following way: In the case that the inventory position in the beginning of the period exceeds the optimal base stock level, the decision maker should not place an order. However, if the inventory position is lower, he should raise the inventory position up to the base stock level if there is enough supply capacity available; if not, he should take advantage of the full supply capacity available for the current order.

In the following theorem we proceed with the characterization of the behavior of the base stock level in relation to the size of ACI. Intuitively, we expect that when we are facing a possible shortage in supply capacity in future periods, we tend towards increasing the base stock level. With this we stimulate the inventory build-up to avoid possible backorders, which would be the probable consequence of capacity shortage. Along the same line of thought, the base stock level is decreasing with higher supply availability revealed by ACI. We confirm these intuitive results in Part 3 of Theorem 3 and illustrate the optimal ordering policy in Figure 3.

Figure 3 Illustration of the optimal ordering policy



We define the first derivative of a function $f(x, z)$ with respect to x as $f'(x, z)$. Also, observe that $\vec{q}_2 \leq \vec{q}_1$ holds if and only if each element of $\vec{q}_1$ is greater than or equal to the corresponding element of $\vec{q}_2$. Part 3 suggests that when $\vec{q}_2 \le \vec{q}_1$, the decision maker has to raise the base stock level $\hat{y}_t(\vec{q}_2)$ over the one that was optimal in the initial setting, $\hat{y}_t(\vec{q}_1)$.

Theorem 3. *For any $\vec{q}_2 \le \vec{q}_1$, the following holds for all t:*

1. $J_t'(y, \vec{q}_2) \le J_t'(y, \vec{q}_1)$ *for all y,*
2. $f_t'(x, \vec{q}_2) \le f_t'(x, \vec{q}_1^{+})$ *for all x,*
3. $\hat{y}_t(\vec{q}_2) \ge \hat{y}_t(\vec{q}_1)$.

We proceed by giving some additional insights into monotone characteristics of the optimal policy. We continue to focus on how the changes in ACI affect the optimal base stock level. In the first case we want to assess whether the base stock level is affected more by the change in supply capacity availability in one of the imminent periods, or is the change in the available capacity in distant periods more significant. Let us define a unit vector e_i with dimensions equal to the dimensionality of the ACI vector (n-dimensional), where its ith component is 1. With vector e_i we can target a particular component of the ACI vector. What we want to know, is see, how the optimal base stock level is affected by taking away η units of supply capacity in period i from now, in comparison with doing the same thing, but one period further in the future. In Part 1 of Theorem 4 we show that taking away a unit of supply capacity given by ACI in period i, affects the optimal base stock level more than taking away a unit of supply capacity, which is available in later periods, $i+1$ and further. This again is in line with intuition. The closer the capacity restriction is to the current period the more we need to take it into account when setting the appropriate base stock level.

Theorem 4. *The following holds for all t:*

1. $\hat{y}_t(\vec{q} - \eta e_i) \ge \hat{y}_t(\vec{q} - \eta e_{i+1})$ *for i=t+1,...,t+n-1.*
2. $\hat{y}_t(\vec{q} - \eta e_i) - \hat{y}_t(\vec{q}) \le \eta$ *for i=t+1,...,t+n.*

This observation leads us to think about what would be a sufficient response if we were to face a capacity limit. What would be an appropriate change in the optimal base stock level? We start with the base scenario in which the available supply capacity is given by initial ACI. Then we impose a tighter capacity restriction in period $t+i$, by lowering the supply capacity $\vec{q}_{t+i}$ by η units. We are interested in the sensitivity of the optimal base stock level to the change in the capacity limit. In Part 2 of Theorem 4, we show that the change in the base stock level should be lower than the change in the capacity limit, in absolute terms. In other words, each unit decrease in available supply capacity revealed by ACI leads to lower or at most equal increase in the optimal base stock level. There is a dampening effect present, and together with result of Part 1 this implies that the exact ACI for distant future period becomes irrelevant to current ordering decision. This is an important result, suggesting there is no benefit in overextending the ACI horizon n. This is desirable both in terms of having a reliable ACI in a practical setting, as well in terms of reducing the complexity of determining the optimal parameters by operating with small n.

3.1. Equivalence of the $n = 0$ model and the model with no ACI

To determine the value of ACI in Section 4, the performance comparison between our ACI model and the model with no ACI is of interest to us. Model with no ACI assumes that supply capacity is uncertain at the time the order is placed. However, our model assumes that for $n = 0$ the realization of supply capacity available for the current order is known to a decision maker at the time when he places an order, while only future supply capacity availability remains uncertain. Therefore there is a concern that comparing the performance of ACI model under information horizon n with the special case of $n = 0$, will not give the full value of ACI. In this Section we show that under the zero lead time situation in a special case of $n = 0$, the decision maker is not better off in comparison with the case where he has no ACI available.

We proceed by rewriting optimal cost function formulation given in (2) for $n = 0$ and $L = 0$:

$$f_t(x_t, q_t) = min_{x_t \leq y_t \leq x_t + q_t}\{C_t(y_t) + \alpha E_{D_t, Q_{t+1}} f_{t+1}(y_t - D_t, q_{t+1})\}, \text{ for } \ 1 \leq t \leq T,$$

where f_t is now only a function of inventory position before ordering x_t and the supply capacity available for the current order q_t.

In the no ACI case, the decision maker has to decide for the order size without knowing what the available supply capacity for the current period will be. Inventory position after ordering y_t only gets updated after the order is actually received, where it can happen that the actual delivery size is less than the order size due to the limited supply capacity. We can therefore write y_t as

$$y_t = x_t + \min\{z_t, q_t\}.$$

This is different to y_t as defined in our ACI model, which already reflects the actual delivery size at the time the order is placed. The optimal cost function formulation for the model with no ACI is given as

$$H_t(x_t, z_t) = \min_{x_t \leq y_t}\{E_{Q_t} C_t(y_t - [z_t - q_t]^+) + \alpha E_{D_t, Q_t} H_{t+1}(y_t - [z_t - q_t]^+ - D_t)\}, \text{ for } \ 1 \leq t \leq T. \quad (4)$$

Observe that the minimization is done for all possible ending inventory positions above the starting inventory position. Since the updating of the inventory position happens before the current period demand needs to be satisfied, it is intuitively clear that by knowing the supply capacity information q_t we cannot come up with a better ordering decision, thus optimal base stock levels and the performance should be the same for both models. The no ACI model given by (4) is a generalization of the model by Ciarallo et al. (1994), where they assume stationarity of demand and capacity (although not stating explicitly that it is required), to cover the settings with non-stationary demand and capacity also. We formalize the above considerations in the following remark.

REMARK 2. *The ACI model given by (2) in the case when $n = 0$ and $L = 0$, and the model with no ACI (4) are equivalent with respect to:*

1. *Optimal base stock level and*
2. *Optimal discounted expected cost.*

We point out that having only q_t, representing currently available supply capacity information, does not help in making better ordering decisions (see also Remark 1). This means that in a zero lead time setting we can choose the $n = 0$ scenario as the base case, since it directly corresponds to the case where we do not have any supply capacity information at all. Based on this observation the value of ACI is defined in (8). In a $L > 0$ setting, employing the information about the available supply capacity for the current order actually improves the performance. Because the order is delivered L periods later, it is beneficial to know to what extent the order will be satisfied already when the order is placed. If the information is received after the order is placed, we step across to ASI model we have mentioned in Section 2.1.

3.2. Link to ADI modeling

In this section we explore the possible similarities between the proposed ACI model and the stream of ADI modeling we have mentioned in the literature review. This is done by constructing the capacitated ADI model, where we want to derive the model structurally equivalent to the our ACI model. If possible, deriving the ADI model with the same optimal base stock levels and the optimal inventory cost, would be the best option. This would give additional insights into the analysis of ACI models, as we would be able to directly implement the solutions and algorithms already proposed in the ADI literature. We complement the formal model construction with an explanation of the conceptual differences between the two models.

Our ADI model assumes that we have two customer types, ones that give their orders N periods ahead, and the customers that order in a usual way, giving their orders for the current period. At the end of the period t, we record the *unobserved* part of demand d_t and the *observed* part of demand o_{t+N}, for the future period $t+N$. Parameter N represents the length of the information horizon over which ADI is available (perfect ADI is assumed). Since ADI for period $t+N$ gets collected through period t, it only gets revealed at the end of the period. Also, we assume that there is a fixed supply capacity Q^c limiting the size of the order in each period. The idea behind the construction of the ADI model is that the capacity Q^c available in each period is used both to cover the observed part of demand and the unobserved part of demand. Since the observed part of demand is modeled as a realization of a non-stationary random variable, the remaining capacity is also a random variable. This suggest that we have a random capacity available to satisfy the unobserved part of demand d_t, which is conceptually similar to our limited stochastic demand ACI inventory model.

To derive a formal ADI model description we start by writing x_t given in (3) in a modified form

$$\tilde{x}_t = \hat{x}_t + \sum_{s=t-L}^{t-1} \tilde{z}_s - \sum_{s=t}^{t+L} o_s, \tag{5}$$

where $\tilde{x}_t$ and $\tilde{z}_s$ represent ADI counterparts to x_t and z_t. Observe that $\tilde{x}_t$ already accounts for the observed part of the lead time demand. By ordering, we raise $\tilde{x}_t$ to the corresponding modified inventory position after ordering $\tilde{y}_t$, given as $\tilde{y}_t = \tilde{x}_t + \tilde{z}_t$, where $\tilde{z}_t \leq Q^c$. Due to the update of the inventory position with observed demand, $\tilde{y}_t$ only needs to cover the remaining, unobserved part of demand. From (5), we can show that at the start of period $t+1$, $\tilde{x}_t$ is updated in the following manner

$$\tilde{x}_{t+1} = \tilde{x}_t + \tilde{z}_t - d_t - o_{t+L+1}.$$

Note that we only know o_{t+L+1} before an order is placed at the start of period $t+1$, if ADI horizon N extends beyond lead time ($N \geq L+1$). If N is shorter, the best we can do is to update $\tilde{x}_{t+1}$ with o_{t+N} we have just collected before the beginning of period $t+1$. This leads to a more trivial version of the model, where it is enough to track the updated modified inventory position, thus the state space is only one-dimensional. For this case the classical results for capacitated systems such as in Federgruen and Zipkin (1986a) apply. However, if ADI horizon extends beyond lead time, we need to track the corresponding demand observations that will affect future orders. These constitute ADI vector $\vec{o}_t = (o_{t+L+1}, \ldots, o_{t+N-1})$ that gets updated at the start of period $t+1$

$$\vec{o}_{t+1} = (o_{t+L+2}, \ldots, o_{t+N}),$$

by including the new information o_{t+N} collected through period t and purging the oldest data point o_{t+L+1}, with which we update $\tilde{x}_{t+1}$. The state space is described by an $N-L$ -dimensional vector $(\tilde{x}_t, \vec{o}_t)$.

The minimal discounted expected cost function of the proposed ADI model, optimizing the cost over a finite planning horizon T from time t onward and starting in the initial state $(\tilde{x}_t, \vec{o}_t)$, can be written as:

$$
\begin{aligned}
f_t(\tilde{x}_t, \vec{o}_t) = \min_{\tilde{x}_t \le \tilde{y}_t \le \tilde{x}_t + Q^c} \{C_t(\tilde{y}_t) + \\
+ \begin{cases} \alpha E_{D_t}[f_{t+1}(\tilde{y}_t - D_t, \vec{o}_{t+1})]\}, & \text{if } T-N-1 \le t \le T, \\ \alpha E_{D_t, O_{t+N}}[f_{t+1}(\tilde{y}_t - D_t, \vec{o}_{t+1})]\}, & \text{if } 1 \le t \le T-N, \end{cases}
\end{aligned}
\tag{6}
$$

where $f_{T+1}(\cdot) \equiv 0$.

Now that we have established the ADI model, we can write the formulations, which establish the relationships between the ACI and ADI model dynamics. For the models to be equivalent in their base stock levels we need to assume that $\tilde{y}_t = y_t$ holds. This is intuitively clear, since both need to cover the same lead time demand. Note that we assume the unobserved part of lead time demand in ADI model is modeled in the same way as ACI model's lead time demand given Section 2.1.

By substituting the relevant ACI dynamics formulations (1) and (3), into the corresponding ADI formulations, we show that ACI model's x_t and z_t are related to their ADI model counterparts $\tilde{x}_t$ and $\tilde{z}_t$ in a following way

$$
\begin{aligned}
x_t &= \tilde{x}_t + o_{t+L}, \\
z_t &= \tilde{z}_t - o_{t+L}.
\end{aligned}
$$

By updating $\tilde{x}_t$ at the start of the period t, we already use some of the total capacity Q^c to cover the observed part of demand o_{t+L}. Now we now only have the remaining capacity $Q^c - o_{t+L}$ to sufficiently raise y_t to cover the unobserved part of lead time demand. This directly relates to having finite capacity q_t, which limits the extent to which we can raise y_t, in ACI model. We can write the following relationship

$$q_t = Q^c - o_{t+L}. \tag{7}$$

By comparing the above ADI model with our ACI model, we first derive the relationship between the two information horizon parameters. Observe that ADI vector $\vec{o}_t$ gives observed demands for $N-L-1$ periods beyond the lead time, which will affect the size of the following $N-L-1$ orders. This corresponds to having ACI for future n periods given by the ACI vector $\vec{q}_t$. ACI and ADI information horizon parameters n and N are therefore related in the following way

$$n = N - L - 1,$$

where we confirm that $N \geq L+1$ has to hold, since we assume $n \geq 0$.

A more important observation is made by substituting ADI dynamics formulations into (2). Comparing the constructed new ADI dynamic programming formulation with the one proposed by (6), shows the inconsistency at the lower bound over which the minimization is made. Instead of looking for the optimal $\tilde{y}_t$ by minimizing the cost function over $\tilde{x}_t \leq \tilde{y}_t \leq \tilde{x}_t + Q^c$, minimization is made over $\tilde{x}_t + o_{t+L} \leq \tilde{y}_t \leq \tilde{x}_t + Q^c$. For a direct equivalence of the ACI model and the ADI model to hold, we always have to be able to raise $\tilde{y}_t$ above $\tilde{x}_t + o_{t+L}$ for all t. This means that the available capacity Q^c should be sufficient to cover at least the observed part of demand. This "negativity issue" also stems directly from (7), where for $q_t \geq 0$, $Q^c - o_{t+L} \geq 0$ has to hold. The fixed capacity therefore has to exceed all possible realizations of the observed part of demand, $Q^c \geq o_t$. Obviously this unrealistic assumption would only hold for large capacities, $Q^c \rightarrow \infty$, which due to (7) leads to $q_t \rightarrow \infty$, and this suggests an uncapacitated system. In an uncapacitated system we would be always able to raise the inventory position high enough to account for the relevant demand realization. We are excluding the possibility that the available capacity is not sufficient to cover the observed part of demand, which has to be allowed in a general ADI model.

Based on these findings we conclude that there are restrictive, even unrealistic, assumptions needed to guarantee a direct equivalence between the capacitated ADI model and the proposed ACI model in terms of optimal base stock levels and optimal costs. However, many of the structural properties of ADI optimal policy hold also in the case of our ACI model. This observation justifies the effort made in studying the ADI literature, as additional similarities could be found between the structural properties and practical implications of the two lines of modeling. An example of this is the fact that the ADI model given by (6) is a specific case of the model introduced by Özer and Wei (2004). Our ADI model is a simplification, since we assume that advance orders are only given N periods in advance, while they allow for additive increase in ADI, by updating the advance orders in each period.

4. Value of ACI

In this section we present the results of the numerical analysis, which was carried out to quantify the value of ACI, and to gain insights into how the value of ACI changes as some of the system parameters change. Numerical calculations were done by solving the dynamic programming formulation given in (2). We limit ourselves to the zero lead-time case ($L = 0$), which effectively reduces the complexity of numerical calculations, but still provides us with the major insights. We (1) introduce the value of ACI as the measure of the relative cost decrease in case when using ACI, over the case when no ACI is available, (2) construct the set of experiments with different demand and capacity patterns. This enables us to describe the influence of average capacity utilization and period-to-period mismatch between the demand and capacity pattern on the inventory cost. At the same time we evaluate the value of ACI and explore the extent of the benefits that can be gained by increasing ACI horizon. (3) Based on a particular demand and capacity pattern we proceed with a more detailed analysis of the influence of the uncertainty of period-to-period demand and capacity, and the changes in the cost structure on the value of ACI.

To identify the value of ACI, we measure the magnitude of inventory cost reduction in the cases where ACI is available for future orders. We define the relative value of ACI for $n > 0$, $\%V_{ACI}$, as the relative difference between the optimal expected cost of managing the system where $n = 0$, and the system where we have an insight into future supply availability $n > 0$:

$$\%V_{ACI}(n > 0) = \frac{f_{n=0} - f_{n>0}}{f_{n=0}}. \tag{8}$$

Based on findings in Section 3.1, summarized in Remark 2, we know that $\%V_{ACI}$ gives the full value of ACI. Thus, measuring the full benefit of using ACI over the no ACI scenario in which the decision maker is facing the uncertain deliveries by his supplier.

We also define the absolute change in the value of ACI, $\triangle V_{ACI}$. With this we measure the extra benefit gained by extending the length of ACI horizon by one time period, from n to $n+1$:

$$\triangle V_{ACI}(n+1) = f_n - f_{n+1}$$

We proceed with constructing a set of four experiments (experiments number 1-4) with different demand and capacity patterns. The remaining parameters are set at the fixed value of: $T = 8, \alpha = 0.99, h = 1, b = 20$, Normal demand and capacity both with a coefficient of variation

(CV) of 0.45. We give a graphical illustration of demand and supply capacity patterns, by plotting the expected demand and expected supply capacity for each of the periods in Figures 4 and 5. The optimal inventory costs and the optimal base stock levels $\hat{y}_{(n=0)}$ for $n = 0$ setting are presented in Table 2, where also the value of ACI, $\%V_{ACI}$, and the absolute change in the value of ACI, $\triangle V_{ACI}$, is given.

Figure 4 Expected demand and capacity pattern, and optimal base stock level $\hat{y}_{(n=0)}$ (a) Exp. 1 (b) Exp. 2.

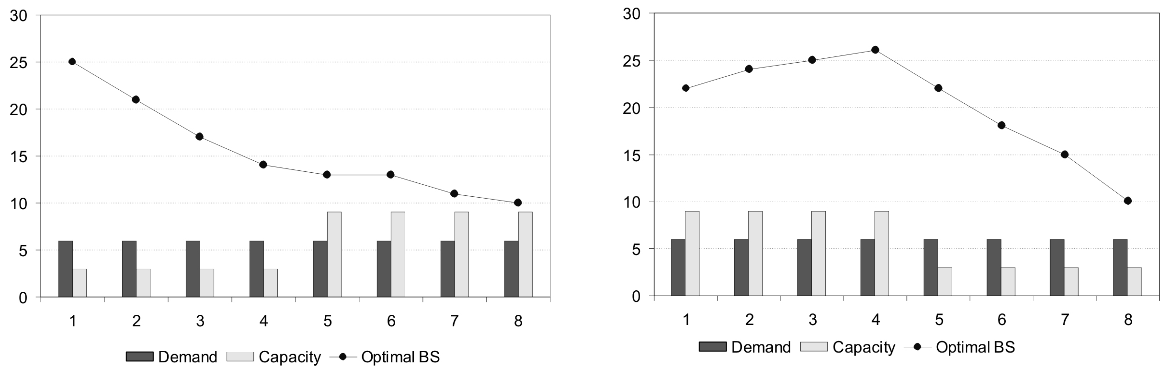
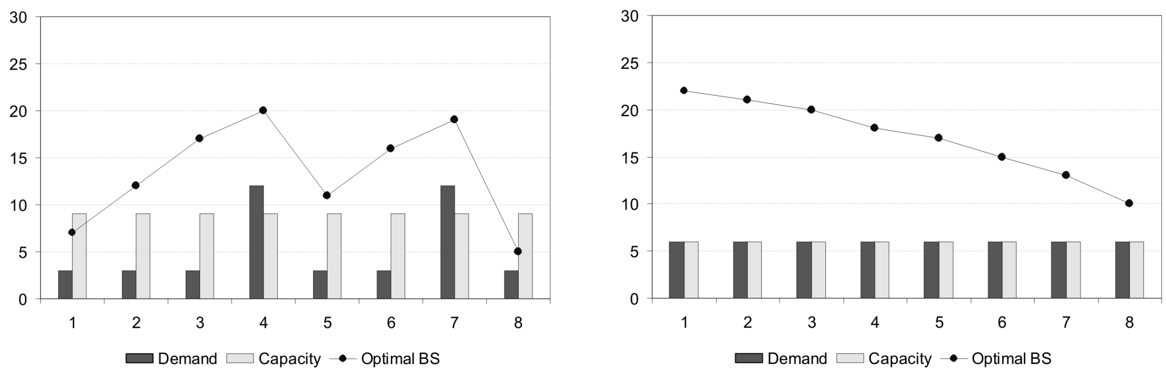


Figure 5 Expected demand and capacity pattern, and optimal base stock level $\hat{y}_{(n=0)}$ (a) Exp. 3 (b) Exp. 4.



Let us first observe the differences in inventory cost between the proposed settings. In general the inventory costs are higher if the supply capacity is highly utilized (experiment number 1, 2 and 4). However, although the average utilization[1] in all three cases is 100%, there are still considerable cost differences. These are mainly due to period-to-period mismatch between demand and supply capacity pattern. In experiment number 1 presented in Figure 4 (a), which represents a kind of of the "worst case" scenario, we are first faced with multiple periods of high demand and inadequate capacity. Therefore the extent of the backorders accumulated in these

[1] Defined as the sum of average demands over the sum of average supply capacities over the whole planning horizon.

first periods is high, thus inventory costs are high. We see that the optimal policy instructs that we raise the base stock levels in the beginning periods. By doing this, we aim to use as much of the available capacity as possible. However, the probability to achieve these target base stock levels is minor, thus we cannot avoid the backorders. If we reverse this setting in experiment number 2 presented in Figure 4 (b), we can use the early excess capacity to build up the necessary inventory to cope with the subsequent capacity shortage. This in turn greatly reduces the cost. We gradually increase the base stock levels as we approach the over-utilized periods. After the peak demand periods, towards the end of the planning horizon, the base stock level drops to the myopic optimal level in the last period[2]. We can further confirm the inventory build up insight by inspecting the experiment number 3 in Figure 5 (a), where we are faced with two demand peaks in periods 4 and 7. To avoid the probable backorders in the two critical and the following periods, the rational thing to do is to pre-build the inventory.

Table 2 Optimal base stock level $\hat{y}_{(n=0)}$, optimal system cost, and the value of ACI

Exp.	t	1	2	3	4	5	6	7	8	n	Cost	$\%V_{ACI}$	$\triangle V_{ACI}$
1	$E[D_t]$	6	6	6	6	6	6	6	6	0	1108.0		
	$E[Q_t]$	3	3	3	3	9	9	9	9	1	1107.6	0.04	0.43
	$\hat{y}_{(n=0)}$	25	21	17	14	13	13	11	10	2	1107.6	0.04	0.02
										3	1107.6	0.04	0.00
										4	1107.6	0.04	0.00
2	$E[D_t]$	6	6	6	6	6	6	6	6	0	279.1		
	$E[Q_t]$	9	9	9	9	3	3	3	3	1	278.3	0.28	0.78
	$\hat{y}_{(n=0)}$	22	24	25	26	22	18	15	10	2	277.6	0.53	0.70
										3	277.2	0.67	0.40
										4	277.0	0.74	0.20
3	$E[D_t]$	3	3	3	12	3	3	12	3	0	78.8		
	$E[Q_t]$	9	9	9	9	9	9	9	9	1	70.6	10.37	8.16
	$\hat{y}_{(n=0)}$	7	12	17	20	11	16	19	5	2	66.7	15.36	3.93
										3	65.7	16.62	0.99
										4	65.4	17.00	0.30
4	$E[D_t]$	6	6	6	6	6	6	6	6	0	477.4		
	$E[Q_t]$	6	6	6	6	6	6	6	6	1	476.6	0.16	0.78
	$\hat{y}_{(n=0)}$	22	21	20	18	17	15	13	10	2	476.2	0.25	0.41
										3	476.0	0.28	0.15
										4	476.0	0.29	0.04

We have recognized the potential settings, where anticipation of supply capacity to demand mismatches, can bring considerable cost reductions. We summarize these conclusions and can give the following conditions that should be fulfilled for anticipation to bring the considerable

[2] In period T, at the end of the planning horizon, the myopic and the optimal solution converge. Since this is the last period, the myopic optimization is optimal as Ciarallo et al. (1994) show for the capacitated single period problem.

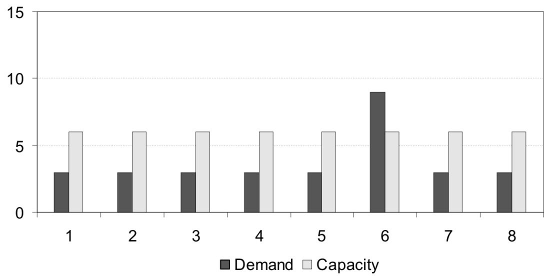
benefits to a decision maker: (1) When there is a mismatch between the demand and supply capacity, meaning that there are time periods when the supply capacity is highly utilized or even over-utilized, but there are also periods when capacity utilization is low. (2) When we can anticipate the possible mismatch in the future, for which we should have some data on the demand and supply capacity probability distributions. (3) When there is enough time and/or excess capacity to build up the inventory to a desired level to avoid backorder accumulation during the capacity shortage.

The relevance of anticipating the future capacity shortages is well established already in a deterministic analysis of the non-stationary capacitated inventory systems. Also in a stochastic variant of this problem, that we are considering, some anticipation is possible through knowing the probability distributions for demand and supply capacity in future periods. In the case of experiment number 4 presented in Figure 5 (b) we are facing a stationary situation, where there is no "deterministic" anticipation. However, knowing that future demand and supply capacity realizations may deviate from their expected values, we raise the base stock levels to account for these uncertainties. In this paper we argue that we can further improve on this anticipative build up by using ACI, and by this we can attain the cost savings.

Looking at the results of Table 2, we now put attention on the cost reductions that can be achieved by using ACI. We see that ACI makes the inventory cost reduction possible, however only in certain cases, while in other the benefits are almost nonexistent. If there is no pre-build opportunity (experiment number 1) we see that the value of ACI is close to 0, no matter what the length of the ACI horizon is. In the case of experiment number 3 we see that by having an insight into next period's available capacity, we can lower the inventory cost by 10.37%, while an additional period of ACI data gives an additional 4.99% cost reduction. However, prolonging the ACI horizon further does not improve the performance greatly. Obviously additional information on future supply conditions can only help in making more effective ordering decisions, therefore we see that $\%V_{ACI}$ increases with the length of the information horizon n. Observe also that $\triangle V_{ACI}$ gets lower, when we increase n. This makes sense intuitively, since knowing the realizations of supply capacity in the near future periods has a higher effect on cost reduction then information on supply limitations in more distant time periods.

We proceed by a close inspection of the influence of the cost structure and the variability of both the demand and supply capacity on the value of ACI. The base scenario is characterized by the following parameters: $T = 6, \alpha = 0.99, h = 1$, discrete uniform distribution is used to model demand and supply capacity, where the expected demand is given as $E[D]_{1..8} = (3, 3, 3, 3, 3, 9, 3, 3)$ and the expected supply capacity as $E[Q]_{1..8} = (6, 6, 6, 6, 6, 6, 6, 6)$. We vary:

Figure 6 Expected demand and capacity pattern for Exp. 5-13.



(1) The cost structure, by changing the backorder cost $b = (1, 20, 100)$ and keeping the inventory holding cost constant at $h = 1$, and (2) the coefficient of variation of demand $CV_D = (0, 0.25, 0.45, 0.65)$ and supply capacity $CV_Q = (0, 0.25, 0.45, 0.65)$, where the CVs do not change through time[3]. The expected demand and supply capacity pattern is presented in Figure 6, and the optimum costs in Table 3.

We see that we have one demand peak in period 6 and five pre-build periods (periods 1-5) we can use to accumulate inventory. Since system's average utilization is 63%, and the peak demand period only occurs after five periods of low demand, both primary conditions, which allow us the build up of inventory, are met: pre-build time and excess capacity. Looking first at the inventory cost, we clearly see that the more volatile the system is (the higher the CV_D and CV_Q are), the worse is the system performance. Changing the cost structure by increasing the backorder cost obviously also increases the inventory cost. When we look at the effect of ACI on inventory costs, we see that the costs can be substantially decreased, even up to almost 70% (experiment number 6). Note again that extending n leads to higher cost savings, but $\triangle V_{ACI}$ is diminishing.

With higher backorder cost there is more potential to decrease the overall cost, but only if we are successful with the inventory build up. Observe this by looking at the value of ACI with regards to b, when we increase CV_Q from experiment number 6 to 8. While in general $\triangle V_{ACI}$ is higher when b is increased, this is not the case with $\%V_{ACI}$. The highest relative cost savings occur for $b = 100$ in experiment number 6, but the opposite is true in experiment number 8. This is due to the fact that when b is high, preventing backorders from occurring is of most importance and this may require a long pre-build phase. In our case there is limited opportunity to accumulate inventory in the first five periods, particularly when we are faced with highly variable supply capacity and demand, therefore the savings through ACI are relatively lower.

[3] Since it is not possible to come up with the exact same CVs for discrete uniform distributions with different means, we give the approximate average CVs for demand and supply capacity distributions with means $E[D]_{1..8}$ and $E[Q]_{1..8}$.

Table 3 Optimal system cost and the value of ACI under varying ACI horizon n

			b	1	20	100	1	20	100	1	20	100
Exp.	CV_D	CV_Q	n	Cost			$\%V_{ACI}$			$\triangle V_{ACI}$		
5	0	0	all n	2.85	2.88	2.88	0.00	0.00	0.00	0.00	0.00	0.00
6	0	0.25	0	3.23	9.44	11.51						
			1	2.99	5.77	6.40	7.47	38.95	**44.39**	0.24	3.68	5.11
			2	2.98	4.15	4.23	7.74	56.08	63.27	0.01	1.62	2.18
			3	2.97	3.82	3.82	7.97	59.56	66.80	0.01	0.33	0.41
			4	2.97	3.79	3.79	8.00	59.90	**67.11**	0.00	0.03	0.04
7	0	0.45	0	5.58	28.88	59.22						
			1	4.60	20.21	48.08	17.48	30.03	18.81	0.98	8.67	11.14
			2	4.57	15.34	40.33	18.09	46.88	31.90	0.03	4.87	7.75
			3	4.46	12.93	36.39	20.14	55.21	38.56	0.11	2.41	3.94
			4	4.45	11.97	34.87	20.31	58.57	41.11	0.01	0.97	1.51
8	0	0.65	0	11.15	77.84	248.87						
			1	8.95	66.12	237.45	**19.75**	15.06	4.59	2.20	11.72	11.42
			2	8.83	58.62	227.39	20.80	24.69	8.63	0.12	7.50	10.06
			3	8.58	54.10	220.59	23.00	30.50	11.36	0.25	4.53	6.80
			4	8.56	51.84	217.10	23.25	33.41	12.77	0.03	2.26	3.49
9	0.25	0.45	0	10.17	41.28	86.60						
			1	9.47	35.19	78.13	6.94	14.76	9.78	0.71	6.09	8.47
			2	9.34	32.04	72.53	8.21	22.39	16.24	0.13	3.15	5.60
			3	9.30	30.51	69.77	8.63	26.08	19.43	0.04	1.52	2.76
			4	9.29	29.94	68.69	8.73	27.46	20.68	0.01	0.57	1.08
10	0.45	0.45	0	15.27	61.50	139.65						
			1	14.75	56.89	133.95	3.37	7.50	4.08	0.51	4.61	5.69
			2	14.65	54.33	130.49	4.07	11.66	6.55	0.11	2.55	3.46
			3	14.62	53.02	128.29	4.24	13.79	8.13	0.03	1.31	2.21
			4	14.61	52.50	127.31	4.28	14.63	8.83	0.01	0.52	0.97
11	0.65	0.45	0	21.16	87.57	224.55						
			1	20.73	84.22	221.12	2.02	3.82	1.53	0.43	3.35	3.43
			2	20.63	82.18	219.22	2.47	6.16	2.37	0.10	2.04	1.90
			3	20.61	81.05	217.90	2.57	7.44	2.96	0.02	1.13	1.32
			4	20.61	80.59	217.20	2.60	7.98	3.27	0.00	0.46	0.70
12	0.25	0.25	0	8.02	22.05	28.98						
			1	7.90	20.28	25.55	1.47	8.06	11.87	0.12	1.78	3.44
			2	7.86	19.20	23.38	2.02	12.96	19.34	0.04	1.08	2.17
			3	7.85	18.84	22.28	2.09	14.58	23.12	0.01	0.36	1.10
			4	7.85	18.78	21.96	2.10	14.86	24.22	0.00	0.06	0.32
13	0.65	0.65	0	24.33	135.97	455.85						
			1	23.53	131.15	450.80	3.28	3.55	1.11	0.80	4.83	5.06
			2	23.29	127.94	447.64	4.26	5.91	1.80	0.24	3.21	3.15
			3	23.22	126.15	445.45	4.56	7.22	2.28	0.07	1.79	2.20
			4	23.20	125.37	444.33	4.63	7.80	2.53	0.02	0.78	1.11

In experiments number 6, 7 and 8, where deterministic demand is assumed, we observe that $\triangle V_{ACI}$ is higher with increasing supply capacity variability. In the case of high CV_Q, we account for the uncertainty of supply capacity by increasing the base stock levels. This leads to too high inventory levels when actual supply capacity realizations are above expected, and to inventory stockouts in the opposite case. However, through ACI, the decision maker is warned beforehand about the possible occurrence of these shortages/stockouts in the future and he can align the base stock level more precisely, depending on the future capacity availability. If we look solely at the effect of increasing demand uncertainty we observe just the opposite. This is clearly seen when CV_D increases both $\%V_{ACI}$ and $\triangle V_{ACI}$ are decreasing, when looking through experiments number 6, 9, 10 and 11. The latter is intuitively clear, since the benefits of a more precise alignment of the base stock level, possible due to revealed ACI, are greatly diminished because the volatile demand causes the inventory position to deviate from the planned level.

To summarize, we have noted that increase in both CV_D and CV_Q leads to higher inventory costs. However, as we have shown, these cost can be significantly decreased by using ACI. We can give a general remark by saying the more chaos there is in supply, the better it is to have some partial (preferably exact) information about the future. The decision maker facing low demand uncertainty on one side, while struggling with high capacity uncertainty on the other, can therefore gain the most from using ACI.

5. Conclusion

In this paper, we have developed a model that incorporates ACI into inventory decision making and explored its effect on making effective ordering decisions within a periodic review inventory planning system facing limited stochastic supply. Based on the proofs that establish the convexity of the relevant cost functions, we are able to show the form of the optimal policy to be a modified base stock policy with a single state-dependent base stock level. Essentially the base stock level depends on realizations of future supply capacities revealed by ACI, and is a decreasing function of the ACI size. We complement this result by showing additional monotone properties of the optimal policy. Another contribution of this work is in establishing a link to advance demand information modelling by derivation of the capacitated ADI model. We show that under certain restrictive assumptions the models are equivalent in their structure. Through this we suggest that there is an interesting overlap between the two research fields.

By means of numerical analysis we develop some additional managerial insights. In particular, we give the following conditions when inventory costs can be decreased through the use ACI: (1) when there is a mismatch between demand and supply capacity, which can be anticipated through ACI, and there exists an opportunity to pre-build inventory in an adequate manner, (2) when uncertainty in future supply capacity is high and ACI is used to lower it effectively, and (3) in the case of high backorder costs, which further emphasizes the importance of avoiding stock outs. Under such circumstances, the companies should pursue establishing long-term contractual agreements, which would encourage ACI sharing. Such relations would bring considerable operational cost savings.

We plan some extensions to the model presented in this paper in our future research. While the proposed model assumes perfect ACI, we want to extend the model to describe the situation where the communicated supply limit might not be completely accurate. This information can be denoted as imperfect ACI. The consequence of ACI not being exact is that there is still some uncertainty in the actual supply capacity availability. This leads to a situation where the inventory position does not reflect the actual realizations of the orders in the pipeline and anticipating the future supply conditions is harder due to the remaining share of the uncertainty. The next step will be a further modification in this direction, where imperfect advance supply information is updated from period to period in a manner in which the accuracy of the information is increased (the variance of the underlying probability distribution is decreased). Finally, we plan to study the model where ACI would be given over an information horizon of uncertain length. This would correspond to the situation where the supplier communicates ACI to the customer, but it does not specify how long the information will be valid. It might hold for many periods, but it can also change unexpectedly. Another possible extension is complementing ACI with possibility of capacity reservations. The customer would assess whether the future supply capacity availability is adequate based on the available ACI, if not, he could take advantage of reserving a certain share of the supplier's capacity in advance and incurring some additional cost of reservation.

Appendix

Preliminaries for Theorem 1

LEMMA 1. *Let $x \in \mathbb{R}^n$, $b \in \mathbb{R}^m$, $A \in \mathbb{R}^{m \times n}$ and $e \in \mathbb{R}^s$. Assume that $g(x,e)$ is convex in x and e. Then the function $f(b,e) := \min_{Ax \leq b} g(x,e) : \mathbb{R}^n \to \mathbb{R}$ is also convex in b and e.*

Proof: Let $0 \leq \theta \leq 1$. Then

$$\begin{aligned}
\theta f(b,e) + (1-\theta) f(\bar{b},\bar{e}) &= \theta \min_{Ax \leq b} g(x,e) + (1-\theta) \min_{Ax \leq x} g(x,\bar{e}) \\
&= \theta g(x_0^*, e) + (1-\theta) g(x_1^*, \bar{e}) \\
&\geq g(\theta(x_0^*, e) + (1-\theta)(x_1^*, \bar{e})) && (9) \\
&\geq \min_{Ax \leq \theta b + (1-\theta)\bar{b}} g(x, \theta e + (1-\theta)\bar{e}) && (10) \\
&= f(\theta(b,e) + (1-\theta)(\bar{b},\bar{e}))
\end{aligned}$$

(9) is due to convexity of g and (10) is because $Ax_0^* \leq b$ and $Ax_1^* \leq \bar{b}$ implies that $A(\theta x_0^* + (1-\theta)x_1^*) \leq \theta b + (1-\theta)\bar{b}$. □

LEMMA 2. *If $J(y,e)$ is convex then $f(x,q,e) = \min_{x \leq y \leq x+q} J(y,e)$ is also convex.*

Proof: Let $h(b,e) := \min_{Ay \leq b} J(y,e)$ where $A = [-1,1]$ and $b = [-x, x+q]$. By Lemma 1, we conclude that $h(b,e)$ is convex. Since $h(b,e) = f(x,q,e)$, f is also convex. □

Proof of Theorem 1: The theorem can be proven by regular inductive arguments and the results of Lemmas 1 and 2. □

Proof of Theorem 2: Convexity results of Theorem 1 directly imply the proposed structure of the optimal policy. □

LEMMA 3. *Let $f(x)$ and $g(x)$ be convex, and x_f and x_g be their smallest minimizers. If $f'(x) \leq g'(x)$ for all x, then $x_f \geq x_g$.*

Proof: Assume for a contradiction that $x_f < x_g$, then $f'(x_g - 1) \geq 0$. This is due to $x_f < x_g$ and is equal to 0 in the extreme case $x_g = x_f + 1$. Also from $f'(x) \leq g'(x)$ we have $f'(x_g - 1) \leq g'(x_g - 1)$. We have $g'(x_g - 1) \geq 0$, which contradicts the definition of x_g being smallest minimizer of $g(\cdot)$. Hence $x_f \geq x_g$. □

Proof of Remark 1: The result directly follows from the results of Theorem 2. □

Proof of Theorem 3: Using the induction argument we first observe that $J_t'(y,\vec{q}_2) \leq J_t'(y,\vec{q}_1)$ holds for $t = T$ since $J_T'(y,\vec{q}) = C_T'(y_t)$. Assuming that it also holds for t we have $\hat{y}_t(\vec{q}_2) \geq \hat{y}_t(\vec{q}_1)$ by using Lemma 3. To prove that this implies $f_t'(x,\vec{q}_2) \leq f_t'(x,\vec{q}_1)$ for t, we first write the optimal cost function $f_t(x,\vec{q})$ using the definition given in (2) and following the results of Theorem 2, as

$$f_t(x,q,\vec{q}) = \begin{cases} J_t(x,\vec{q}), & \hat{y}_t(\vec{q}) \leq x, \\ J_t(\hat{y}_t(\vec{q}),\vec{q}), & \hat{y}_t(\vec{q}) - q \leq x < \hat{y}_t(\vec{q}), \\ J_t(x+q,\vec{q}), & x < \hat{y}_t(\vec{q}) - q, \end{cases} \tag{11}$$

where q denotes available supply capacity for the current order. Observe also that we later use $f_t(x,\vec{q}) = E_Q f_t(x,q,\vec{q})$, unless noted otherwise. From this, the definition of the first difference and the convexity of J_t proven in Theorem 1, we can write $f_t'(x,\vec{q})$ in the following manner

$$f_t'(x,q,\vec{q}) = \begin{cases} \geq 0, & \hat{y}_t(\vec{q}) \leq x, \\ = 0, & \hat{y}_t(\vec{q}) - q \leq x < \hat{y}_t(\vec{q}), \\ < 0, & x < \hat{y}_t(\vec{q}) - q. \end{cases} \tag{12}$$

Further analysis considers all possible inventory positions before ordering x_t with regards to both optimal base stock levels $\hat{y}_t(\vec{q}_1)$ and $\hat{y}_t(\vec{q}_2)$; and available supply capacity for the current order, denoted as q_1 and q_2. Out of the possible nine cases in total, we can eliminate four cases due to the two conditions: $\hat{y}_t(\vec{q}_2) \geq \hat{y}_t(\vec{q}_1)$ and $q_1 \leq q_2$. Thus we only need to closely analyze the remaining five cases, showing that $f_t'(x, \vec{q}_2) \leq f_t'(x, \vec{q}_1)$ holds.

Case 1: If $(x \geq \hat{y}_t(\vec{q}_2)) \wedge (x \geq \hat{y}_t(\vec{q}_1))$, which holds on the interval $x \geq \hat{y}_t(\vec{q}_2)$ for any combination $q_1 \leq q_2$, then $f_t'(x, \vec{q}_1) = J_t'(x, \vec{q}_1)$ and $f_t'(x, \vec{q}_2) = J_t'(x, \vec{q}_2)$ from (11) and since Part 1 states $J_t'(y, \vec{q}_2) \leq J_t'(y, \vec{q}_1)$, we have $f_t'(x, \vec{q}_2) \leq f_t'(x, \vec{q}_1)$.

Case 2: If $(\hat{y}_t(\vec{q}_2) - q_2 \leq x < \hat{y}_t(\vec{q}_2)) \wedge (x \geq \hat{y}_t(\vec{q}_1))$ or equivalently $\hat{y}_t(\vec{q}_2) - q_2 \leq x < \hat{y}_t(\vec{q}_2)$ for $q_1 \leq q_2 \leq \hat{y}_t(\vec{q}_2) - \hat{y}_t(\vec{q}_1)$, then $f_t'(x, \vec{q}_1) \geq 0$ and $f_t'(x, \vec{q}_2) = 0$ from (12), thus $f_t'(x, \vec{q}_2) \leq f_t'(x, \vec{q}_1)$.

Case 3: If $(x < \hat{y}_t(\vec{q}_2) - q_2) \wedge (x \geq \hat{y}_t(\vec{q}_1))$ or equivalently $\hat{y}_t(\vec{q}_1) \leq x < \hat{y}_t(\vec{q}_2) - q_2$ for $q_1 \leq q_2 \leq \hat{y}_t(\vec{q}_2) - \hat{y}_t(\vec{q}_1)$, then $f_t'(x, \vec{q}_1) \geq 0$ and $f_t'(x, \vec{q}_2) < 0$ from (11), thus $f_t'(x, \vec{q}_2) \leq f_t'(x, \vec{q}_1)$.

Case 4: If $(\hat{y}_t(\vec{q}_2) - q_2 \leq x < \hat{y}_t(\vec{q}_2)) \wedge (\hat{y}_t(\vec{q}_1) - q_1 \leq x < \hat{y}_t(\vec{q}_1))$, which holds on $\hat{y}_t(\vec{q}_2) - q_2 \leq x < \hat{y}_t(\vec{q}_1)$ for $q_2 > \hat{y}_t(\vec{q}_2) - \hat{y}_t(\vec{q}_1)$ and any $q_1 \leq q_2$, then $f_t'(x, \vec{q}_1) = 0$ and $f_t'(x, \vec{q}_2) = 0$ from (11), thus $f_t'(x, \vec{q}_2) \leq f_t'(x, \vec{q}_1)$.

Case 5: If $(x < \hat{y}_t(\vec{q}_2) - q_2) \wedge (\hat{y}_t(\vec{q}_1) - q_1 \leq x < \hat{y}_t(\vec{q}_1))$, which holds on $\hat{y}_t(\vec{q}_1) - q_1 \leq x < \hat{y}_t(\vec{q}_2) - q_2$ for $q_2 > \hat{y}_t(\vec{q}_2) - \hat{y}_t(\vec{q}_1)$ and any $q_1 \leq q_2$, then $f_t'(x, \vec{q}_1) = 0$ and $f_t'(x, \vec{q}_2) < 0$ from (11), thus $f_t'(x, \vec{q}_2) \leq f_t'(x, \vec{q}_1)$.

Going from t to $t-1$ we conclude the induction argument by showing that $f_t'(x, \vec{q}_2) \leq f_t'(x, \vec{q}_1)$ implies $J_{t-1}'(y, \vec{q}_2) \leq J_{t-1}'(y, \vec{q}_1)$. Using the definition we write $J_{t-1}'(y, \vec{q}_2) = C_{t-1}'(y) + \alpha E_{D_{t-1}, Q_{t+n}} f_t(x_2, q_2, \vec{q}_2)$, where going backwards x_2 is updated from y and order size z_t is limited by the available supply capacity given by q_2. Taking into account that we have condition $\vec{q}_2 \leq \vec{q}_1$ at $t-1$, this implies $q_2 \leq q_1$ at t, and we can conclude that $x_2 \leq x_1$ has to hold. Thus, $f_t'(x_2, \vec{q}_2) \leq f_t'(x_1, \vec{q}_2)$ also holds directly from convexity and we can write $C_{t-1}'(y) + \alpha E_{D_{t-1}, Q_{t+n}} f_t(x_2, \vec{q}_2) \leq C_{t-1}'(y) + \alpha E_{D_{t-1}, Q_{t+n}} f_t(x_1, \vec{q}_2) \leq C_{t-1}'(y) + \alpha E_{D_{t-1}, Q_{t+n}} f_t(x_1, \vec{q}_1) = J_{t-1}'(y, \vec{q}_1)$, where the second inequality is due to induction argument $f_t'(x, \vec{q}_2) \leq f_t'(x, \vec{q}_1)$. With this we have shown $J_{t-1}'(y, \vec{q}_2) \leq J_{t-1}'(y, \vec{q}_1)$ holds and the proof is completed. $\square$

LEMMA 4. *For any $\vec{q}$ and $\eta > 0$ and all t, we have:*

1. $J_t'(x - \eta, \vec{q}) \leq J_t'(x, \vec{q} - \eta e_1)$,
2. $\hat{y}_t(\vec{q} - \eta e_1) - \hat{y}_t(\vec{q}) \leq \eta$,
3. $f_t'(x - \eta, \vec{q}) \leq f_t'(x, \vec{q} - \eta e_1)$.

Proof: For all x and t,

$$J_t'(x - \eta, \vec{q}) = C_t'(x - \eta) + \alpha E_{D_t, Q_{t+n+1}} f_{t+1}'(x - \eta - D_t, q_{t+1}, \vec{q}_{t+1})$$

$$\leq C_t'(x) + \alpha E_{D_t,Q_{t+n+1}} f_{t+1}'(x-\eta-D_t, q_{t+1}, \vec{q}_{t+1})$$
$$= J_t'(x, \vec{q}-\eta e_1). \quad (13)$$

The inequality is due to convexity of $C_t(x)$ and this proves Part 1 assuming $f_t'(x-\eta,\vec{q}) \leq f_t'(x,\vec{q}-\eta e_1)$ holds. Notice that the smallest minimizer of $J_t'(x-\eta,\vec{q})$ is $\hat{y}_t(\vec{q})+\eta$. This together with Lemma 3 implies $\hat{y}_t(\vec{q}-\eta e_1) \leq \hat{y}_t(\vec{q})+\eta$, which proves Part 2. We continue to prove $f_t'(x-\eta,\vec{q}) \leq f_t'(x,\vec{q}-\eta e_1)$ for all x and t. To do so, we use (14) and (12), and consider 9 possible cases:

Case 1: If $\hat{y}_t(\vec{q}) \leq x-\eta$ and $\hat{y}_t(\vec{q}-\eta e_1) \leq x$ then $f_t'(x-\eta,\vec{q}) = J_t'(x-\eta,\vec{q}) \leq J_t'(x,\vec{q}-\eta e_1) = f_t'(x,\vec{q}-\eta e_1)$. The inequality follows from (13).

Case 2: If $\hat{y}_t(\vec{q}) \leq x-\eta$ and $\hat{y}_t(\vec{q}-\eta e_1)-q \leq x < \hat{y}_t(\vec{q}-\eta e_1)$ is not possible since $x < \hat{y}_t(\vec{q}-\eta e_1) \leq \hat{y}_t(\vec{q})+\eta$.

Case 3: If $\hat{y}_t(\vec{q}) \leq x-\eta$ and $x < \hat{y}_t(\vec{q}-\eta e_1)-q$ is not possible since $x < \hat{y}_t(\vec{q}-\eta e_1)-q \leq \hat{y}_t(\vec{q})+\eta-q$.

Case 4: If $\hat{y}_t(\vec{q})-q \leq x-\eta < \hat{y}_t(\vec{q})$ and $\hat{y}_t(\vec{q}-\eta e_1) \leq x$ then $f_t'(x-\eta,\vec{q}) = 0 \leq f_t'(x,\vec{q}-\eta e_1)$.

Case 5: If $\hat{y}_t(\vec{q})-q \leq x-\eta < \hat{y}_t(\vec{q})$ and $\hat{y}_t(\vec{q}-\eta e_1)-q \leq x < \hat{y}_t(\vec{q}-\eta e_1)$ then $f_t'(x-\eta,\vec{q}) = 0 = f_t'(x,\vec{q}-\eta e_1)$.

Case 6: If $\hat{y}_t(\vec{q})-q \leq x-\eta < \hat{y}_t(\vec{q})$ and $x < \hat{y}_t(\vec{q}-\eta e_1)-q$ is not possible since $x < \hat{y}_t(\vec{q}-\eta e_1)-q \leq \hat{y}_t(\vec{q})+\eta-q$.

Case 7: If $x-\eta < \hat{y}_t(\vec{q})-q$ and $\hat{y}_t(\vec{q}-\eta e_1) \leq x$ then $f_t'(x-\eta,\vec{q}) < 0 \leq f_t'(x,\vec{q}-\eta e_1)$.

Case 8: If $x-\eta < \hat{y}_t(\vec{q})-q$ and $\hat{y}_t(\vec{q}-\eta e_1)-q \leq x < \hat{y}_t(\vec{q}-\eta e_1)$ then $f_t'(x-\eta,\vec{q}) < 0 = f_t'(x,\vec{q}-\eta e_1)$.

Case 9: If $x-\eta < \hat{y}_t(\vec{q})-q$ and $x < \hat{y}_t(\vec{q}-\eta e_1)-q$ then $f_t'(x-\eta,\vec{q}) = J_t'(x-\eta+q,\vec{q}) \leq J_t'(x+q,\vec{q}-\eta e_1) = f_t'(x,\vec{q}-\eta e_1)$. The inequality follows from (13). With this we conclude the proof of Part 3. □

Proof of Theorem 4: Part 3 of Lemma 4 implies that $J_t'(y,\vec{q}-\eta e_1) = C_t'(y) + \alpha E_{D_t,Q_{t+n+1}} f_{t+1}'(y-\eta-D_t, q_{t+1}, \vec{q}_{t+1}) \leq C_t'(y) + \alpha E_{D_t,Q_{t+n+1}} f_{t+1}'(y-D_t, q_{t+1}, \vec{q}_{t+1}-\eta e_1) = J_t'(y,\vec{q}-\eta e_2)$ for all t. Hence for all t,

$$J_t'(y,\vec{q}-\eta e_i) \leq J_t'(y,\vec{q}-\eta e_{i+1}) \quad (14)$$

is true for $i=1$. Now assume for an induction argument that (14) is true for i, then Part 1 for i follows from Lemma 3. Next, we show that Part 1 for i and the induction argument imply $f_t'(x,\vec{q}-\eta e_i) \leq f_t'(x,\vec{q}-\eta e_{i+1})$. To do so, we use (11) and consider 3 cases:

Case 1: If $x \geq \hat{y}_t(\vec{q} - \eta e_{i+1})$, then $f_t'(x, \vec{q} - \eta e_{i+1}) = J_t'(x, \vec{q} - \eta e_{i+1}) \geq J_t'(x, \vec{q} - \eta e_i) = f_t'(x, \vec{q} - \eta e_i)$, where the inequality is due to (14).

Case 2: If $\hat{y}_t(\vec{q} - \eta e_{i+1}) - q \leq x < \hat{y}_t(\vec{q} - \eta e_i)$, then $f_t'(x, \vec{q} - \eta e_{i+1}) \geq 0$. This is the case since on the interval $\hat{y}_t(\vec{q} - \eta e_{i+1}) - q \leq x < \hat{y}_t(\vec{q} - \eta e_{i+1})$ it holds $f_t'(x, \vec{q} - \eta e_{i+1}) = 0$, however on $\hat{y}_t(\vec{q} - \eta e_{i+1}) < x \leq \hat{y}_t(\vec{q} - \eta e_i)$, $f_t'(x, \vec{q} - \eta e_{i+1}) \geq 0$. Here $\hat{y}_t(\vec{q} - \eta e_{i+1}) \leq \hat{y}_t(\vec{q} - \eta e_i)$ holds from (14) and Lemma 3. For $f_t'(x, \vec{q} - \eta e_i) \leq 0$, due to $f_t'(x, \vec{q} - \eta e_i) = 0$ on interval $\hat{y}_t(\vec{q} - \eta e_i) - q \leq x < \hat{y}_t(\vec{q} - \eta e_i)$ and $f_t'(x, \vec{q} - \eta e_i) \leq 0$ on interval $\hat{y}_t(\vec{q} - \eta e_{i+1}) - q \leq x < \hat{y}_t(\vec{q} - \eta e_i) - q$. $\hat{y}_t(\vec{q} - \eta e_{i+1}) - q \leq \hat{y}_t(\vec{q} - \eta e_i) - q$ also holds from (14) and Lemma 3.

Case 3: If $x < \hat{y}_t(\vec{q} - \eta e_{i+1}) - q$, then $f_t'(x, \vec{q} - \eta e_{i+1}) = J_t'(x + q, \vec{q} - \eta e_{i+1}) \geq J_t'(x + q, \vec{q} - \eta e_i) = f_t'(x, \vec{q} - \eta e_i)$, where the inequality is due to (14).

From (2) and the cases above, we have $J_t'(y, \vec{q} - \eta e_{i+1}) = C_t'(y) + \alpha E_{D_t, Q_{t+n+1}} f_{t+1}'(y - D_t, q_{t+1}, \vec{q}_{t+1} - \eta e_i) \leq C_t'(y) + \alpha E_{D_t, Q_{t+n+1}} f_{t+1}'(y - D_t, q_{t+1}, \vec{q}_{t+1} - \eta e_{i+1}) = J_t'(y, \vec{q} - \eta e_i)$. This completes the induction argument and the proof of (14) and Part 1. □

The proof of Part 2 follows directly from Part 1 and Lemma 4 for $i > 1$ because $\hat{y}_t(\vec{q} - \eta e_i) - \hat{y}_t(\vec{q}) \leq \hat{y}_t(\vec{q} - \eta e_{i-1}) - \hat{y}_t(\vec{q}) \leq \cdots \leq \hat{y}_t(\vec{q} - \eta e_1) - \hat{y}_t(\vec{q}) \leq \eta$. This proof was inspired by Özer and Wei (2003). □

Proof of Remark 2: For the proof we refer the reader to Jakšič (2008).

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